Chapter 10 - Day 3

FTC: if $f(x)$ is a continuous function and $A(x) = \int_a^x f(t)\,dt$ then

$$A'(x) = \frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)$$

and if $F(x)$ is any antiderivative of $f(x)$ then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Ex: Compute the derivative of $F(x)$ if $F(x)=\int_2^x (t^4+t^3+t+9)\,dt$

*FTC - part 1

$$F'(x)=\frac{d}{dx}\left(\int_2^x (t^4+t^3+t+9)\,dt\right)$$
$$=\boxed{x^4+x^3+x+9}$$

Ex: Compute the derivative of

$$g(s)=\int_5^s \frac{8}{\sqrt{u^2+u+2}}\,du$$

*FTC - part 1

$$g'(s)=\frac{d}{ds}\int_5^s \frac{8}{\sqrt{u^2+u+2}}\,du=\boxed{\frac{8}{\sqrt{s^2+s+2}}}$$

Ex: Suppose $f(x) = \int_1^x \sqrt{t^2 - 7t + 12.25}\, dt$.

For what value of x does $f'(x)$ equal 0?

find $f'(x)$ – use FTC part 1

$$f'(x) = \sqrt{x^2 - 7x + 12.25} = 0$$

$$x^2 - 7x + 12.25 = 0$$

$$x = \frac{7 \pm \sqrt{49 - 4(1)(12.25)}}{2(1)}$$

$$= \frac{7 \pm 0}{2} = \boxed{\frac{7}{2}}$$

Ex: Find the value of x at which

$F(x) = \int_3^x (t^8 + t^6 + t^4 + t^2 + 1)\, dt$ takes

its minimum on the interval [3,100]. The value of x that gives a minimum of F(x) is ____ .

find F'(x) – FTC part 1

$F'(x) = x^8 + x^6 + x^4 + x^2 + 1 > 0$

thus F(x) is always increasing

so minimum occurs when $\boxed{x=3}$.

Ex: Find the value of x at which $G(x) = \int_{-5}^{x} (|t|+2)\,dt$ takes its maximum on $[-5, 100]$. Where is the maximum of $G(x)$?

find $G'(x)$ - FTC part I

$G'(x) = |x| + 2 > 0$ for all x

thus $G(x)$ is always increasing

So maximum occurs at $\boxed{x = 100}$.

Ex: Evaluate $\int_0^5 (t^2+1)\,dt$

*FTC - part 2

$$\int_0^5 (t^2+1)\,dt = \left(\frac{1}{3}t^3 + t\right)\Big|_0^5$$

$$= \left(\frac{1}{3}\cdot 5^3 + 5\right) - \left(\frac{1}{3}\cdot 0^3 + 0\right)$$

$$= \frac{140}{3} - 0 = \boxed{\frac{140}{3}}$$

Ex: Evaluate $\int_{-7}^{-5} \left(\frac{1}{t}\right)^2 dt$

*FTC - part 2

$$\int_{-7}^{-5} t^{-2}\,dt = \left(\frac{1}{-1}t^{-1}\right)\Big|_{-7}^{-5}$$

$$= -(-5)^{-1} - {}^{-}(-7)^{-1}$$

$$= \frac{1}{5} - \frac{1}{7} = \frac{7}{35} - \frac{5}{35} = \boxed{\frac{2}{35}}$$

Ex: Evaluate $\int_0^2 e^x \, dx$ *FTC part 2

$$\int_0^2 e^x \, dx = e^x \Big|_0^2 = e^2 - e^0 = \boxed{e^2 - 1}$$

Ex: Evaluate $\int_{-6}^{12} |t| \, dt$

$$\int_{-6}^{12} |t| \, dt = \int_{-6}^{0} -t \, dt + \int_0^{12} t \, dt$$

$$= \left(\frac{-1}{2}t^2\right)\Big|_{-6}^{0} + \left(\frac{1}{2}t^2\right)\Big|_0^{12}$$

$$= [0 - (-18)] + [72 - 0]$$

$$= \boxed{90}$$

Ex: Evaluate $\int_2^5 \left(3u^5 + \frac{7}{u}\right) du$

$$= \int_2^5 3u^5\,du + \int_2^5 \frac{7}{u}\,du$$

$$= \left(\frac{3}{6}u^6\right)\Big|_2^5 + 7(\ln|u|)\Big|_2^5$$

$$= \frac{15625}{2} - 32 + 7(\ln|5| - \ln|2|)$$

$$= \boxed{7780.5 + 7\ln 5 - 7\ln 2}$$